How 95% Confidence Interval (CI) of HPC QPF Absolute Error (AE) and CI of HPC QPF are computed

Background

HPC produces a suite of deterministic 6-hour Quantitative Precipitation Forecasts (QPFs) out through three days. These forecasts represent the forecasters' best effort at predicting the most likely amount of QPF. These forecasts have been produced for over 40 years and verification statistics show a steady, gradual improvement. The skill of these forecasts varies significantly with season, weather regime, forcing function, and forecast time. For example, skill in forecasting a synoptic scale overrunning event is usually better than convective events.

While these forecasts have proven to be useful as they are, they offer no information concerning the skill of individual forecasts. Many users have expressed a need for an objective way of assessing the likely success of a particular forecast.

Introduction

All QPFs have uncertainty. The availability of ensemble forecasts have allowed forecasters to better assess the uncertainty of a particular model run. By running an ensemble of models with differing physics or initial conditions, forecasters have a better feel for the sensitivity of a particular model run to initial conditions or model physics and can therefore better assess the level of confidence they can place in a particular model run. Unfortunately, it is difficult for forecasters to subjectively use the ensemble output to quantify this uncertainty.

This project is an attempt to objectively quantify the level of confidence that is justified in a particular HPC subjective QPF forecast by relating errors in HPC QPFs to ensemble spread. In general, the larger the spread, the greater will be the uncertainty of a particular forecast.

Overview

Several parameters available from the Short Range Ensemble Forecasts (SREFs) were compared to the errors in HPC forecasts. While there was little correlation between HPC QPF absolute errors (AE) and SREF spreads for 500mb height, 850mb temperature, and 850 and 700mb relative humidity, the best correlation was found to be with the spread of the SREF QPF.

We then derived a relationship between the SREF QPF spread at a given forecast time and gridpoint to the AE in the HPC QPF for that point using linear regression. The Quantitative Precipitation Estimates (QPEs) used for verification were obtained from the National Precipitation Verification Unit (NPVU). The relationship obtained is applied to the current HPC 6-hourly QPF to determine the estimated error in the forecast. The 95% confidence intervals of HPC QPF and HPC QPF AE are also calculated.

This is an ongoing project and is still in its early stages. The results are posted for evaluation by potential users. The techniques will continue to undergo refinements based on verification results and user feedback.

Methodology

The first step was to find a relationship between the desired quantity, the absolute error (AE) of the HPC forecast, and a known quantity such as the spread from the SREF. The purpose of this approach is to quantify the uncertainty in the manually produced HPC QPF. Our experimental study indicates that HPC QPF AE is highly correlated with ensemble QPF spread. The linear regression of HPC QPF AE on ensemble QPF spread can be written in the form:

$$\underbrace{|\mathrm{RFC_QPE}_{ij} - \mathrm{HPC_QPF}_{ij}|}_{\text{“AE}_{ij}\text{”}} = b0_{ij} + b1_{ij}\,\mathrm{SP}_{ij},$$

where RFC_QPE is the observed precipitation (i.e., ground truth), HPC_QPF is the HPC precipitation forecast, b0 is the intercept, b1 is the slope, and SP is the ensemble QPF spread. The subscripts i and j address the horizontal position of each grid point.

For a given set of n_{ij} grid points (SP_{ijk}, AE_{ijk}), it is possible to find the intercept $b0_{ij}$ and slope $b1_{ij}$ that minimize sum of squared errors for multiple times addressed by index k. The regression is done with k varying and i, j fixed. The solution set is

$$b1_{ij} = \frac{S(SP*AE)_{ij}}{SS(SP)_{ij}} \quad \text{and} \quad b0_{ij} = \overline{AE_{ij}} - b1_{ij} * \overline{SP_{ij}}$$

where S(SP*AE) is the sum of cross products (i.e., $\Sigma\ (SP_{ijk} - \overline{SP_{ij}})(AE_{ijk} - \overline{AE_{ij}})$), SS(SP) is the sum of squares for SP (i.e., $\Sigma\ (SP_{ijk} - \overline{SP_{ij}})^2$), $\overline{AE}$ is the mean AE, and $\overline{SP}$ is the mean SP.

Now we want to estimate a future value of AE_{ij} given the ensemble spread SP_{ij} that is within, or at least near, the range of SP's in our data.

The general form of a confidence interval (CI) [e.g., Wilks, 1995; Steel et al., 1997] for the predicted AE at SPo is given by

$$CI(pred.AE_{ijk} at SP_{ijk}) = b0_{ij} + b1_{ij} SPo_{ijk} \pm t_{ij}\sqrt{MSE_{ij}\left[1 + \frac{1}{n_{ij}} + \frac{(SPo_{ijk} - \overline{SP_{ij}})^2}{SS(SP)_{ij}}\right]} \qquad (1)$$

where SPo is the SP value for the individual point we are trying to predict, n is the number of data points, MSE is the mean squared error (i.e., $MSE_{ij} = s^2_{AE \cdot SP} = [SS(AE)_{ij} - b1_{ij} * S(SP*AE)_{ij}]/[n_{ij} - 2]$; Its square root is sometimes referred to as the *standard error of estimate* or the *standard deviation of AE at a fixed SP*), and t is the appropriate percentile of the t distribution with degrees of freedom (df) equal to the error degrees of freedom, n–2 for a simple linear regression. For example, if we are computing

95% prediction interval, we let t=1.960 for df=∞ … t=2.086 for df=20, etc. When we compute the minimum (i.e., lower boundary of the confidence interval, b0 + b1*SPo – t$\sqrt{\ }$) and maximum (i.e., upper boundary of the confidence interval, b0 + b1*SPo + t$\sqrt{\ }$), we can say we are 95% confident that HPC QPF with SP=SPo will have AE between the minimum value and the maximum value.

In addition, we can estimate 95% CI of HPC QPF by solving the equation (1) for the HPC QPF. Equation (1) can be re-written as the following:

$$CI(pred.\,|\,RFC_QPE_{ijk} - HPC_QPF_{ijk}\,|\,atSP_{ijk}) = b0_{ij} + b1_{ij}SPo_{ijk} \pm t_{ij}\sqrt{MSE_{ij}\left[1 + \frac{1}{n_{ij}} + \frac{(SPo_{ijk} - \overline{SP_{ij}})^2}{SS(SP)_{ij}}\right]}$$

------- (2)

After moving the HPC_QPF term to the right-hand side and rearranging the equation, CI of HPC QPF can be written in the form:

$$CI(pred.real_precip_{ijk}) = HPC_QPF_{ijk} \pm \left\{|\,b0_{ij} + b1_{ij}SPo_{ijk}\,| + t_{ij}\sqrt{MSE_{ij}\left[1 + \frac{1}{n_{ij}} + \frac{(SPo_{ijk} - \overline{SP_{ij}})^2}{SS(SP)_{ij}}\right]}\right\}$$

------- (3)

Choosing the positive (negative) sign in equation 3 yields the maximum (minimum) expected precipitation for the upper (lower) CI bound associated with the *HPC QPF*, SP_0, and *t* value. While the level of confidence associated with the *t* value is appropriate for the AE CI estimates, the same degree of confidence may not apply to HPC QPF CI estimates (in particular, degrees of confidence lower than 95%) because the largest possible interval is selected for equation 3.

In order to construct the CI, we usually assume the deviations of the observed AE's from the true fitted line (i.e., true errors, ε_{ijk}) satisfy the following assumptions.

1. They all have mean 0.
2. They all have the same variance.
3. They are uncorrelated.
4. They are normally distributed.

To test our data sets satisfy the above assumptions, the estimates of ε (the deviations of the observed AE's from the fit line; residuals) were computed and examined. First, it was demonstrated that the methodology used for the first step approach satisfy the assumptions 1 and 4. Second, all the MSEs (estimates of the variance of ε) computed for HPC_QPF categorized subsets were not the same. Especially, the MSEs computed for

the wet QPF categorized subsets (QPF≥0.01 inch) were much greater than the MSE for dry QPF subset (0≤QPF<0.01 inch). To address different MSE causing problems (overestimate / underestimate) for dry and wet precipitations, we performed the following experiments for CI computations. The new regression model equation parameters (i.e., slope, intercept, MSE, number of data points, mean spread, sum of squares for spread, etc.) derived using new stratification methodology were applied to CI computations and then the CI forecast results were evaluated.

(1) Method 0 (Dry/All Regressions)
- For Dry QPF (i.e., QPF = 0) CI forecast: Apply the regression model equation parameters derived using the previous year data of 0 ≤ QPF < 0.01 inch.
- For Wet QPF (i.e., QPF > 0) CI forecast: Apply the regression model equation parameters derived using the previous year data of QPF ≥ 0.

(2) Method 1 (Dry/Wet Regressions)
- Dry QPF (i.e., QPF = 0): Apply the regression model equation parameters derived using the previous year data of 0 ≤ QPF < 0.01 inch.
- Wet QPF (i.e., QPF > 0): Apply the regression model equation parameters derived using the previous year data of QPF > 0.

(3) Method 2 (Dry/Light/ModerateHeavy Regressions)
- Dry QPF (i.e., QPF = 0): Apply the regression model equation parameters derived using the previous year data of 0 ≤ QPF < 0.01 inch.
- Light QPF (i.e., 0 < QPF < 0.1 inch): Apply the regression model equation parameters derived using the previous year data of 0 < QPF < 0.1 inch.
- Moderate/Heavy QPF (i.e., QPF ≥ 0.1 inch): Apply the regression model equation parameters derived using the previous year data of QPF ≥ 0.1 inch.

(4) Method 3 (Power Transforming Regressions)
- Dry QPF (i.e., QPF = 0): Apply the regression model equation parameters derived using the previous year data of 0 ≤ QPF < 0.01 inch.
- Wet QPF (i.e., QPF > 0): Apply the regression model equation parameters obtained from the logarithmically transformed data for QPF > 0.

(5) Method 4 (15 QPF Categorized Regressions)
- Dry QPF (QPF = 0): Apply the regression model equation parameters derived using the previous year data of 0 ≤ QPF < 0.01 inch.
- Wet QPF 1 (0.00 < QPF < 0.10): Apply the regression model equation parameters derived using the previous year data of 0 < QPF < 0.10 inch.
- Wet QPF 2 (0.10 ≤ QPF < 0.25): Apply the regression model equation parameters derived using the previous year data of 0 < QPF < 0.25 inch.
- Wet QPF 3 (0.25 ≤ QPF < 0.50): Apply the regression model equation parameters derived using the previous year data of 0 < QPF < 0.50 inch.
- Wet QPF 4 (0.50 ≤ QPF < 0.75): Apply the regression model equation parameters derived using the previous year data of 0 < QPF < 0.75 inch.
- Wet QPF 5 (0.75 ≤ QPF < 1.00): Apply the regression model equation parameters derived using the previous year data of 0 < QPF < 1.00 inch.
- Wet QPF 6 (1.00 ≤ QPF < 1.25): Apply the regression model equation parameters derived using the previous year data of 0 < QPF < 1.25 inch.

- Wet QPF 7 ($1.25 \leq QPF < 1.50$): Apply the regression model equation parameters derived using the previous year data of $0 < QPF < 1.50$ inch.
- Wet QPF 8 ($1.50 \leq QPF < 1.75$): Apply the regression model equation parameters derived using the previous year data of $0 < QPF < 1.75$ inch.
- Wet QPF 9 ($1.75 \leq QPF < 2.00$): Apply the regression model equation parameters derived using the previous year data of $0 < QPF < 2.00$ inch.
- Wet QPF 10 ($2.00 \leq QPF < 2.50$): Apply the regression model equation parameters derived using the previous year data of $0 < QPF < 2.50$ inch.
- Wet QPF 11 ($2.50 \leq QPF < 3.00$): Apply the regression model equation parameters derived using the previous year data of $0 < QPF < 3.00$ inch.
- Wet QPF 12 ($3.00 \leq QPF < 4.00$): Apply the regression model equation parameters derived using the previous year data of $0 < QPF < 4.00$ inch.
- Wet QPF 13 ($4.00 \leq QPF < 5.00$): Apply the regression model equation parameters derived using the previous year data of $0 < QPF < 5.00$ inch.
- Wet QPF 14 ($5.00 \leq QPF <$ infinite): Apply the regression model equation parameters derived using the previous year data of $0 < QPF <$ infinite.

Result

Method 4 (15 QPF Categorized Regressions) was proven to be the best methodology satisfying high hit rate and reasonable confidence interval range. As of March 29, 2004, Method 4 implementation was done for real time confidence interval forecasts for the HPC QPF and HPC QPF absolute error. The verification statistics for Method 4 to address reliability and consistency are now available upon request.

Upgrade 1 (09/20/2004)

We developed an algorithm that is to combine Method 0 and Method 4. When the MSE of Method 4 for each HPC QPF category is greater (less) than the threshold value of $[QPF_{max}/t_value]^2$, the regression parameters derived from Method 0 (Method 4) are applied to the computation of CI forecasts. The verification statistics indicate improvements in the CI sizes (significantly reduced CI size in light rain ranges), while showing ignorable changes in hit rates. Since September 20, 2004, this method has been implemented to the real time CI forecasts.

Upgrade 2 (12/13/2006)

On December 13, 2006, new scheme was implemented to 1) deliver the CI forecast products approximately five hours earlier than previously, 2) adapt more quickly to changes in the operational SREF, and 3) use information more relevant to the current weather regime. The differences between old and new schemes are summarized in the following table.

Previous Scheme	New Scheme
Compute Regression parameters each season and then apply to CI computations	Compute Regression parameters each day using most recent 3 month data and then apply to CI computations
Use 09z and 21z cycle SREF	Use 03z and 15z cycle SREF

Ongoing work

We are making these products available for users for preliminary evaluation and hope they will provide feedback to the HPC. We are also verifying these forecasts to determine their accuracy, calibration, and reliability.

We expect to improve the reliability of the product by increasing the amount of data used in deriving the relationship between SREF spread and possibly modifying the techniques used to apply these relationships to the HPC forecast. We will also be modifying the display of this data based on user feedback.